RR BOLT STRENGTH CALCULATIONS

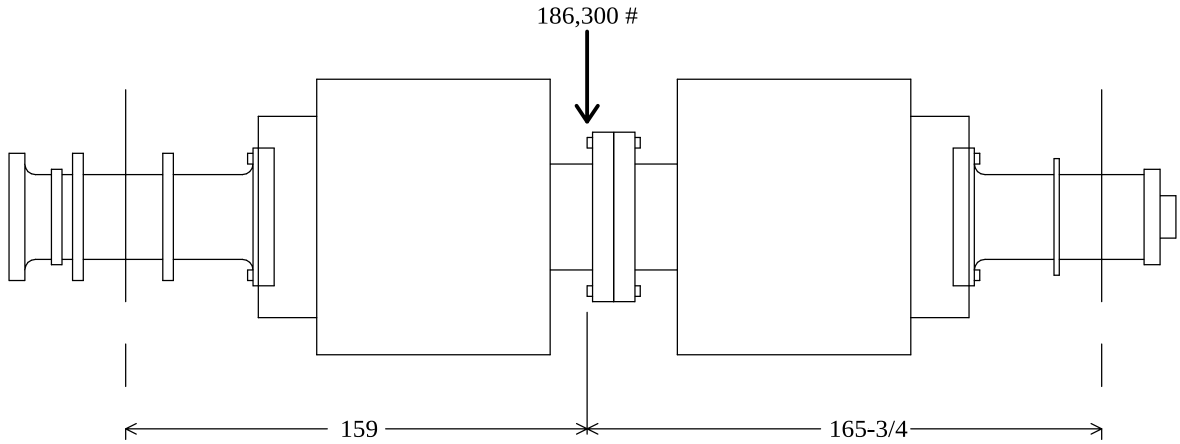
Douglas K. Smithman, P.E.
5/31/2007

Given: Reversing Rougher Armature Assembly dimensions and weights from drawing E-18605.

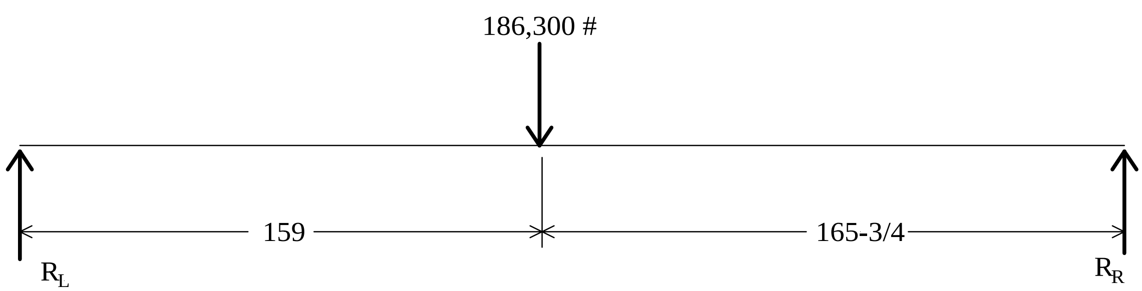
Find: Evaluate the adequacy of the bolt in terms of fatigue.

Solution: The joint is designed to transmit torque through 16 pins. Each pin is retained by a 2 x 12 ASTM A449-Type 1 bolt with 8 pitch threads.

1. Determine the reaction loads and stresses.



Relative locations are shown above. The CG is offset due to the larger coupling and journal on the drive end.



$$\sum F = 0$$

$R_L + R_R - 186{,}300 = 0$
$R_L = 186{,}300 - R_R = 186{,}300 - 91{,}214 = 95{,}086$ #

$$\sum M_{RL} = 0$$

$159(186{,}300) - (159+165.75)R_R = 0$

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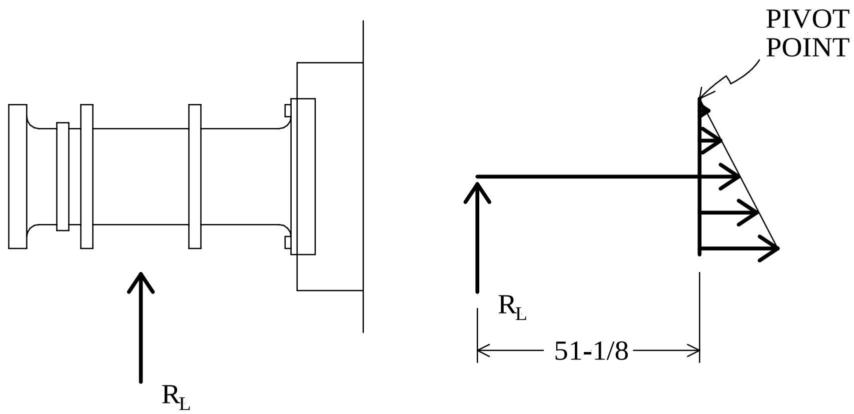
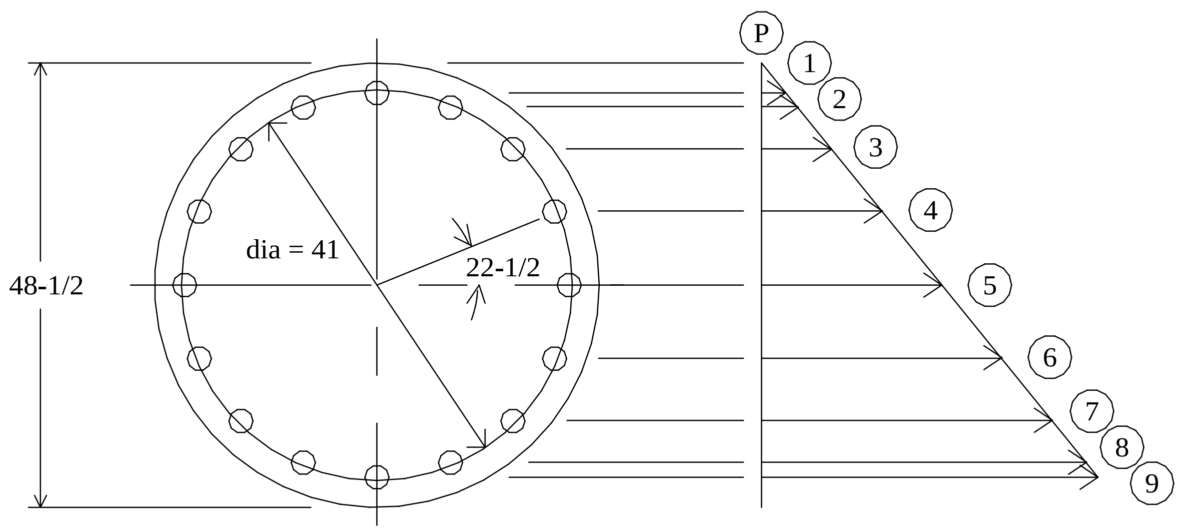
$$R_R = \frac{159(186{,}300)}{324.75} = 91{,}214\#$$

Each bolted flange is essentially being pried off of its commutator by the action of the assembly weight pushing down and the bearing resisting that force. These forces create moments about the top corner of the flange. The stub shaft is being treated as rigid compared to the bolts.



Each bolt will experience its maximum reaction load when it is at the bottom. When it is in this position, there are 9 horizontal reaction planes in the joint. Top and bottom have one bolt each and the seven intermediate planes have two bolts each. If the flange is detailed, the reaction planes look like this.



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The distances from the pivot point to each plane are:

d_1 = ½(48.5-41) =	3.75 inches
d_2 = 48.5/2 - sin(22.5*3)*20.5 =	5.31 "
d_3 = 48.5/2 - sin(22.5*2)*20.5 =	9.75 "
d_4 = 48.5/2 - sin(22.5)*20.5 =	16.40 "
d_5 = 48.5/2 =	24.25 "
d_6 = 48.5/2 + sin(22.5)*20.5 =	32.10 "
d_7 = 48.5/2 + sin(22.5*2)*20.5 =	38.75 "
d_8 = 48.5/2 + sin(22.5*3)*20.5 =	43.19 "
d_9 = 48.5 - ½(48.5-41) =	44.75 "

2. Determine the alternating and mean stresses

Each distance and reaction combination forms two sides of a triangle ad the triangles are similar. Therefore the forces are proportional to the distances. If the maximum reaction force, F_9 is defined as F, then the others can be stated in terms of F.

F_1 = 3.75/44.75 *F =	0.08F
F_2 = 5.31/44.75 *F =	0.12F
F_3 = 9.75/44.75 *F =	0.22F
F_4 = 16.40/44.75 *F =	0.37F
F_5 = 24.25/44.75 *F =	0.54F
F_6 = 32.10/44.75 *F =	0.72F
F_7 = 38.75/44.75 *F =	0.87F
F_8 = 43.19/44.75 *F =	0.97F
F_9 = 44.75/44.75 *F =	1.00F

Now F may be determined by summing the moments.

$$\sum M_P = 0$$

$$51.125(95{,}086) - d_1F_1 - d_2F_2 - d_3F_3 - d_4F_4 - d_5F_5 - d_6F_6 - d_7F_7 - d_8F_8 - d_9F_9 = 0$$

51.125(95,086) - 3.75*0.08F - 5.31*0.12F - 9.75*0.22F - 16.40*0.37F - 24.25*0.54F - 32.10*0.72F - 38.75*0.87F - 43.19*0.97F - 44.75*1.00F = 0

$F_{alt\text{-}max}$ = 51.125 * 95,086 / 165.7 = 29,338 #

This is the maximum axial force on any given bolt.

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The minimum is at F_1 and is:

$F_{alt-min}$ = 0.08 * 29,338 = 2,347 #

This is the minimum axial force on any given bolt.

Susceptibility to fatigue will be determined with a modified Goodman diagram. Reference Fundamentals of Machine Design, 3rd Ed by Joseph Shigley for procedures.

3. Determine the endurance limit.

The endurance limit for rotating beam specimens is, S'_e, = $0.50S_{ut}$ for an ultimate strength, S_{ut} <= 200kpsi, which this is. Therefore:

S'_e, = 0.50 * 90,000 = 45,000 psi

This is an ideal value based on laboratory testing. It must be modified to account for the actual specimen.

$S_e = k_a k_b k_c k_e S'_e$

k_a is a surface factor that derates the specimen from the highly polished surfaces that are laboratory tested. For machined steel at S_{ut} ,= 90,000 psi; $k_a = 0.75$

k_b accounts for reduced endurance in larger specimens. For a 2” bolt, $k_b = 0.75$

k_c is the reliability factor that accounts for the distribution of strengths in actual samples. For a 99% reliability, $k_c = 0.81$

k_a derates for stress concentrations due to discontinuities in the specimen. For a grade 5 fastener with cut threads, $k_e = 1/3.8 = 0.26$

So the endurance limit for these bolts is:

S_e = 0.75*0.75*0.81*0.26*45,000 = 5,331 psi

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4. Determine the stresses

The stress range and stress amplitude may then be calculated. The bolt is a 2 inch with 8 pitch threads. The stress area is 2.7665 in^2.

$\sigma_{alt\text{-}max} = F_{alt\text{-}max} / A = 29{,}338/2.7665 = 10604$ psi

$\sigma_{alt\text{-}min} = F_{alt\text{-}min} / A = 2{,}347/2.7665 = 848$ psi

$\sigma_{range} = \sigma_{alt\text{-}max} - \sigma_{alt\text{-}min} = 10604 - 848 = 9{,}756$ psi

$\sigma_{amp} = (\sigma_{alt\text{-}max} - \sigma_{alt\text{-}min})/2 = (10604 - 848)/2 = 4{,}878$ psi

In addition to this alternating stress, there is a preload stress. Assume the bolts are torqued to 75% of their yield strength. Then:

$\sigma_{mean} = 0.75 * 58{,}000$ psi $= 43{,}500$ psi

$\sigma_{max} = \sigma_{mean} + \sigma_{amp} = 43{,}500 + 4{,}878 = 48{,}378$ psi

5. Plot a modified Goodman diagram to illustrate the results. The top heavy line defines the maximum stress for these bolts. The point defined by σ_{mean} and σ_{max} should lie below that line. If it doesn't then this indicates susceptibility to fatigue failure.

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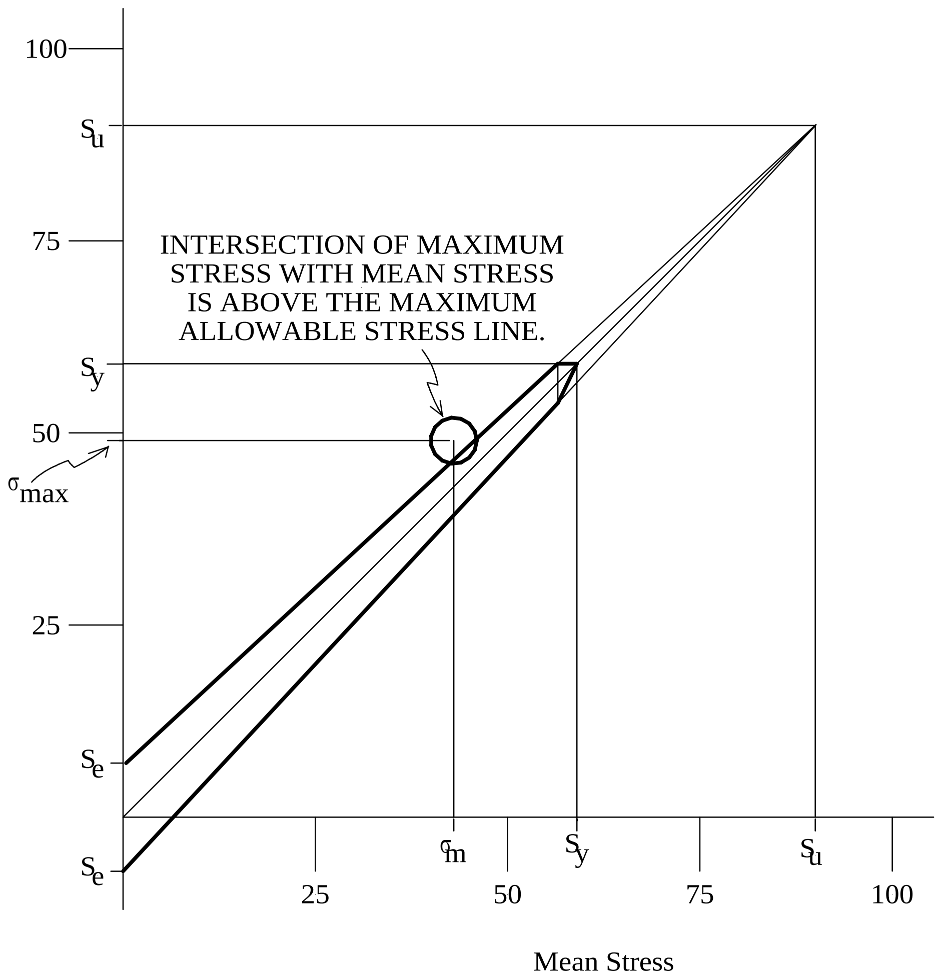

6. Suppose an ASTM A354, Grade BD bolt is used in lieu of the current ASTM A449 bolt. Then the ultimate strength would be 150,000 psi and

S'_e, = 0.50 * 150,000 = 75,000 psi

None of the modification factors would change so the endurance limit would be:

S_e = 0.75*0.75*0.81*0.26*75,000 = 8,884 psi

The modified Goodman diagram for this condition is plotted below. It shows that a higher strength bolt doesn't change the outcome; the bolt is still susceptible to fatigue failures!

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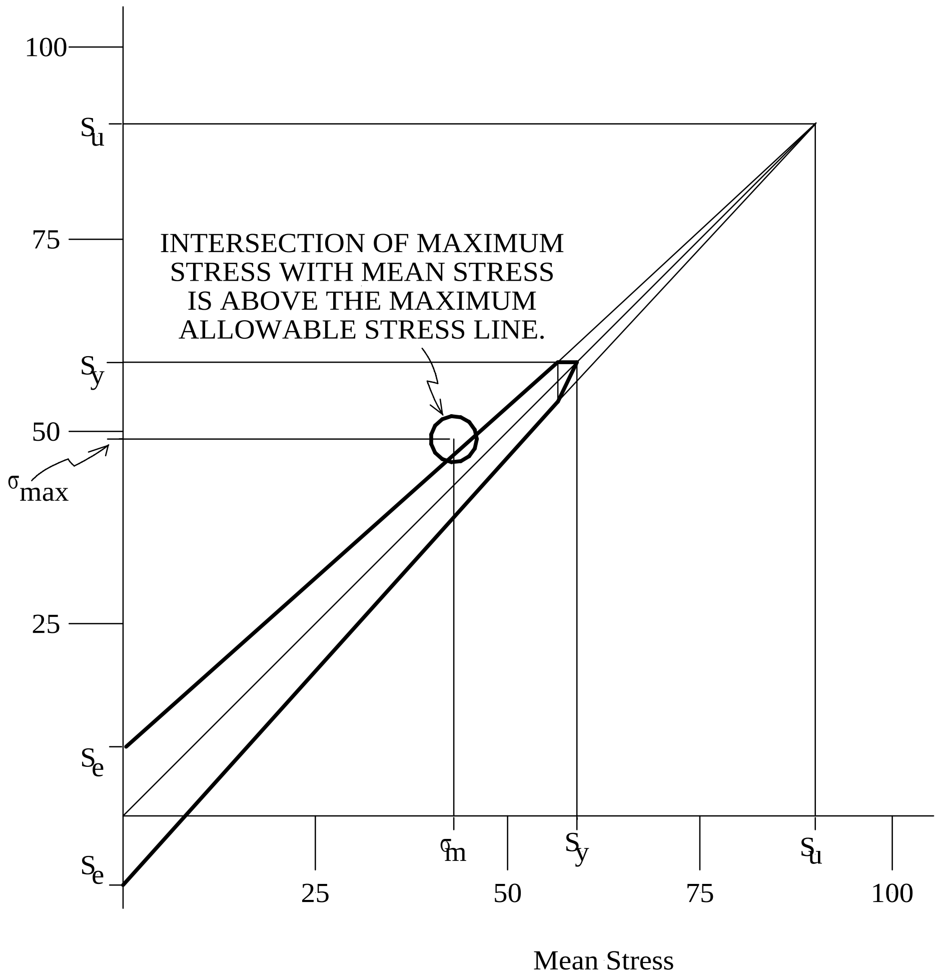

7. Results

<u>These bolts will be subject to fatigue failures. This is true even if higher strength bolts are used.</u> The only potential solution is to reduce the pre-load sufficiently to bring σ_{max} below the maximum stress of 47,000 psi. This means the preload must be below:

47,000 - 5,726 = 41,274 or roughly 40,000 psi.

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8. What if the threads are rolled instead of cut? This will change the endurance limit by changing k_c. For rolled threads on an SAE Grade 5 or 8 fastener, k_f = 3.0. Therefore:

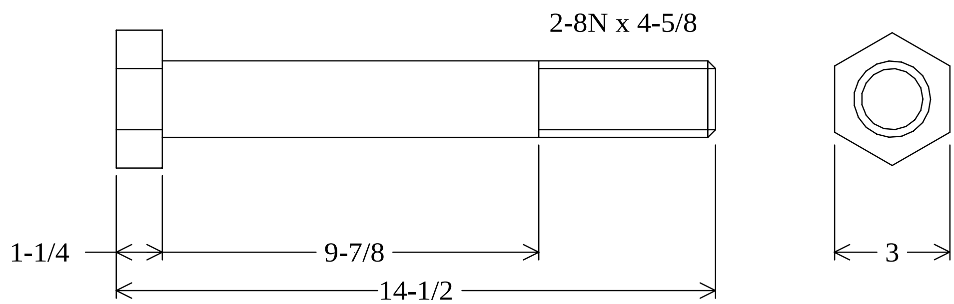
$k_c = 1/3.0 = 0.33$

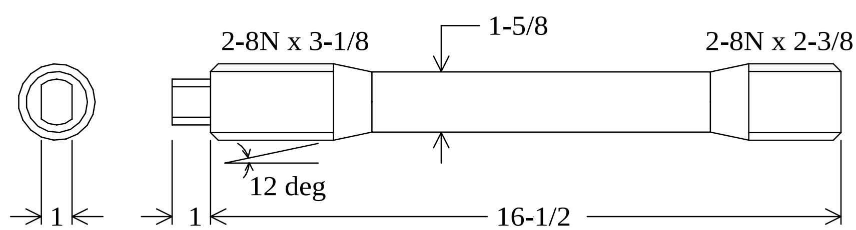
and the new endurance limit is:

S_e = 5,331 * 0.33/0.26 = 6,766 psi. for rolled threads. A quick examination of the modified Goodman plots for this new S_e shows that it will make almost no difference in the location of the Maximum Stress line.

Rolled threads versus cut threads improve the situation slightly but do not eliminate the fatigue problem.

9. A joint diagram is developed to compare both bolt designs. Both are shown below. Note that these are not manufacturing drawings, they are simply for reference.





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The stiffness of each fastener will be calculated. At 75% of S_y and at 38,000 psi of stress. The second value is to keep the fastener threads below a stress level that will make fatigue a concern. The spring constant will be determined, then the preload, then the deflection and finally the maximum load (preload + alternating stress).

For the existing bolt, the diameter is 2.0 inches and the active length is 9.875 inches.

<u>Existing at S_y = 43,500psi</u>

$$k = \frac{F}{\delta} = \frac{AE}{l} = \frac{\pi d^2}{4} \bullet \frac{E}{l} = \frac{\pi (2.0)^2}{4} \bullet \frac{30(10^6)}{9.875} = 9.5(10^6)\, lb/in$$

$$P = A\sigma = \frac{\pi d^2 \sigma}{4} = \frac{\pi}{4}(2.0)^2 (43{,}500) = 136{,}659\, psi$$

$$\delta = \frac{P}{k} = \frac{136{,}659}{9.5(10^6)} = 0.014\, inches$$

$$P_{\max} = 136{,}659 + 29{,}338 = 165{,}997\, lb$$

<u>Existing at S_y = 38,000psi</u>

$$k = \frac{F}{\delta} = \frac{AE}{l} = \frac{\pi d^2}{4} \bullet \frac{E}{l} = \frac{\pi (2.0)^2}{4} \bullet \frac{30(10^6)}{9.875} = 9.5(10^6)\, lb/in$$

$$P = A\sigma = \frac{\pi d^2 \sigma}{4} = \frac{\pi}{4}(2.0)^2 (38{,}000) = 119{,}381\, psi$$

$$\delta = \frac{P}{k} = \frac{119{,}381}{9.5(10^6)} = 0.013\, inches$$

$$P_{\max} = 119{,}381 + 29{,}338 = 148{,}719\, lb$$

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For the new stud, the diameter is 1.563 inches and the active length is 10.99 inches. The affect of the tapered sections on each end will be neglected.

New at S_y = 43,500psi

$$k = \frac{F}{\delta} = \frac{AE}{l} = \frac{\pi d^2}{4} \bullet \frac{E}{l} = \frac{\pi (1.563)^2}{4} \bullet \frac{30(10^6)}{10.99} = 5.2(10^6) lb/in$$

$$P = A\sigma = \frac{\pi d^2 \sigma}{4} = \frac{\pi}{4}(1.563)^2 (43,500) = 83,464 psi$$

$$\delta = \frac{P}{k} = \frac{83,464}{5.2(10^6)} = 0.016 inches$$

$$P_{\max} = 83,464 + 29,338 = 112,802 lb$$

New at S_y = 38,000psi

$$k = \frac{F}{\delta} = \frac{AE}{l} = \frac{\pi d^2}{4} \bullet \frac{E}{l} = \frac{\pi (1.563)^2}{4} \bullet \frac{30(10^6)}{10.99} = 5.2(10^6) lb/in$$

$$P = A\sigma = \frac{\pi d^2 \sigma}{4} = \frac{\pi}{4}(1.563)^2 (38,000) = 72,910 psi$$

$$\delta = \frac{P}{k} = \frac{72,910}{5.2(10^6)} = 0.014 inches$$

$$P_{\max} = 72,910 + 29,338 = 102,248 lb$$

These values have been used to develop the following joint diagrams. The stiffness of the flange is assumed to be 4 times the stiffness of the original fastener in all of these diagrams.

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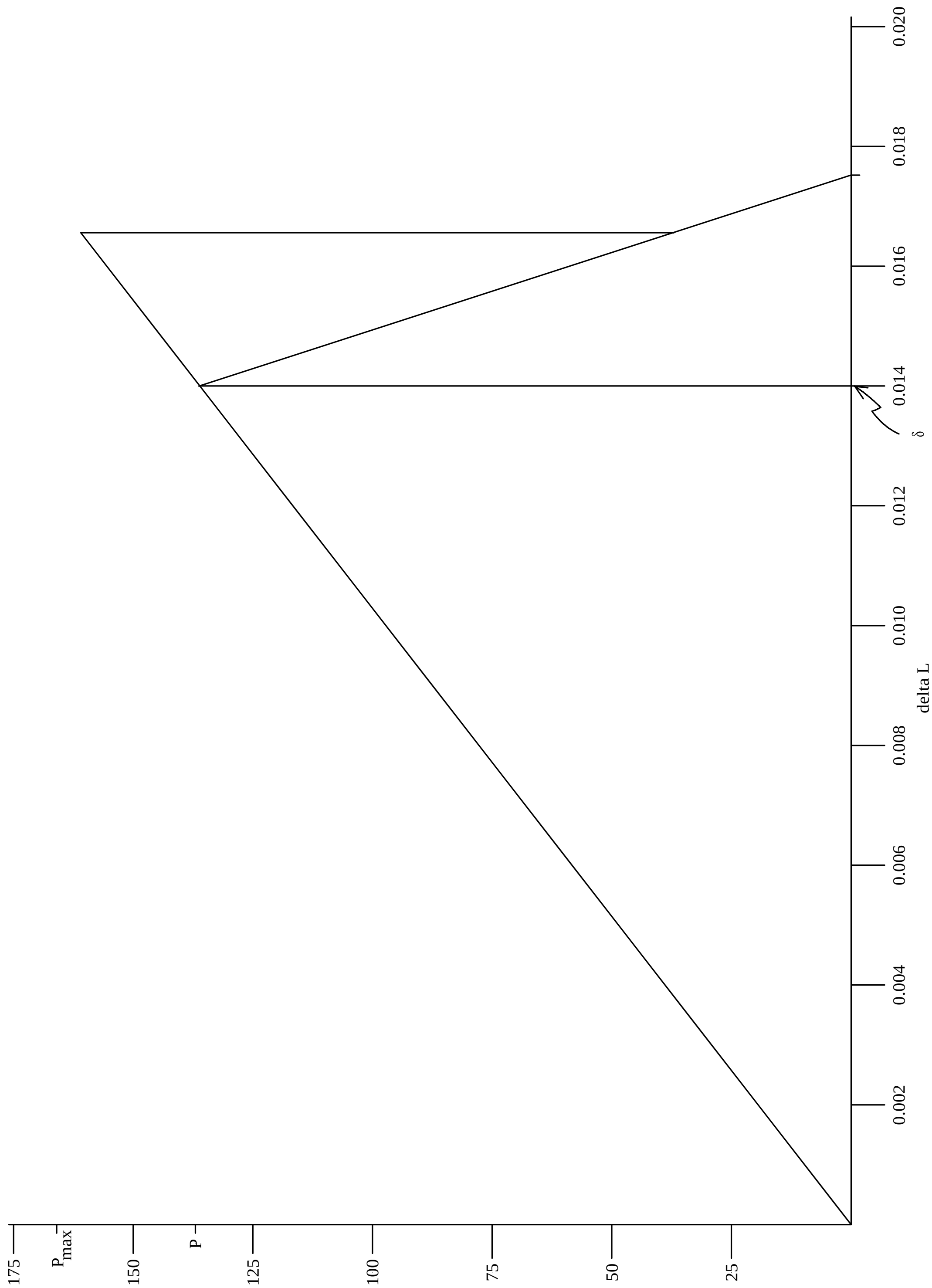
Existing at S_y = 43,500psi



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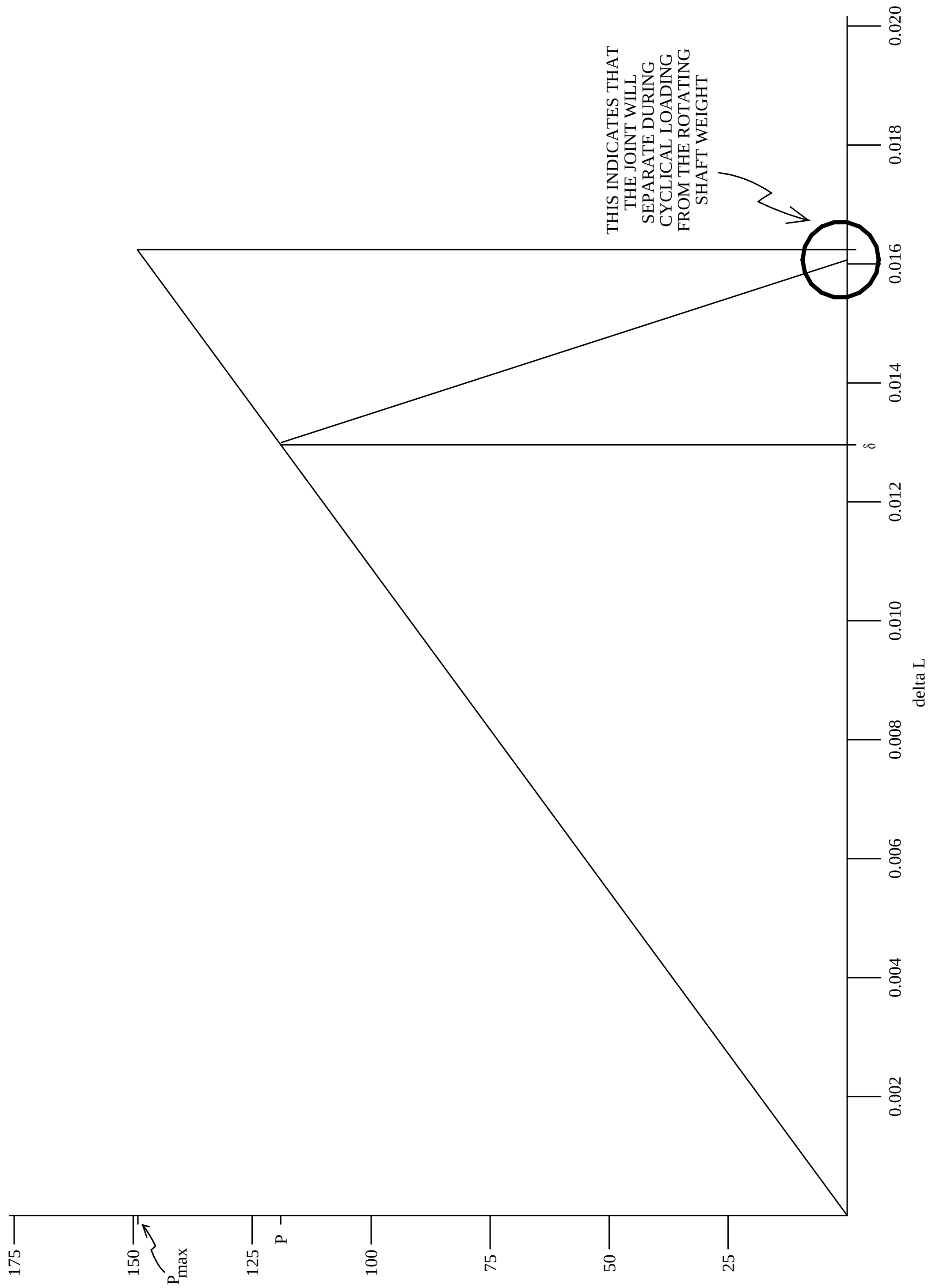
Existing at S_y = 38,000psi



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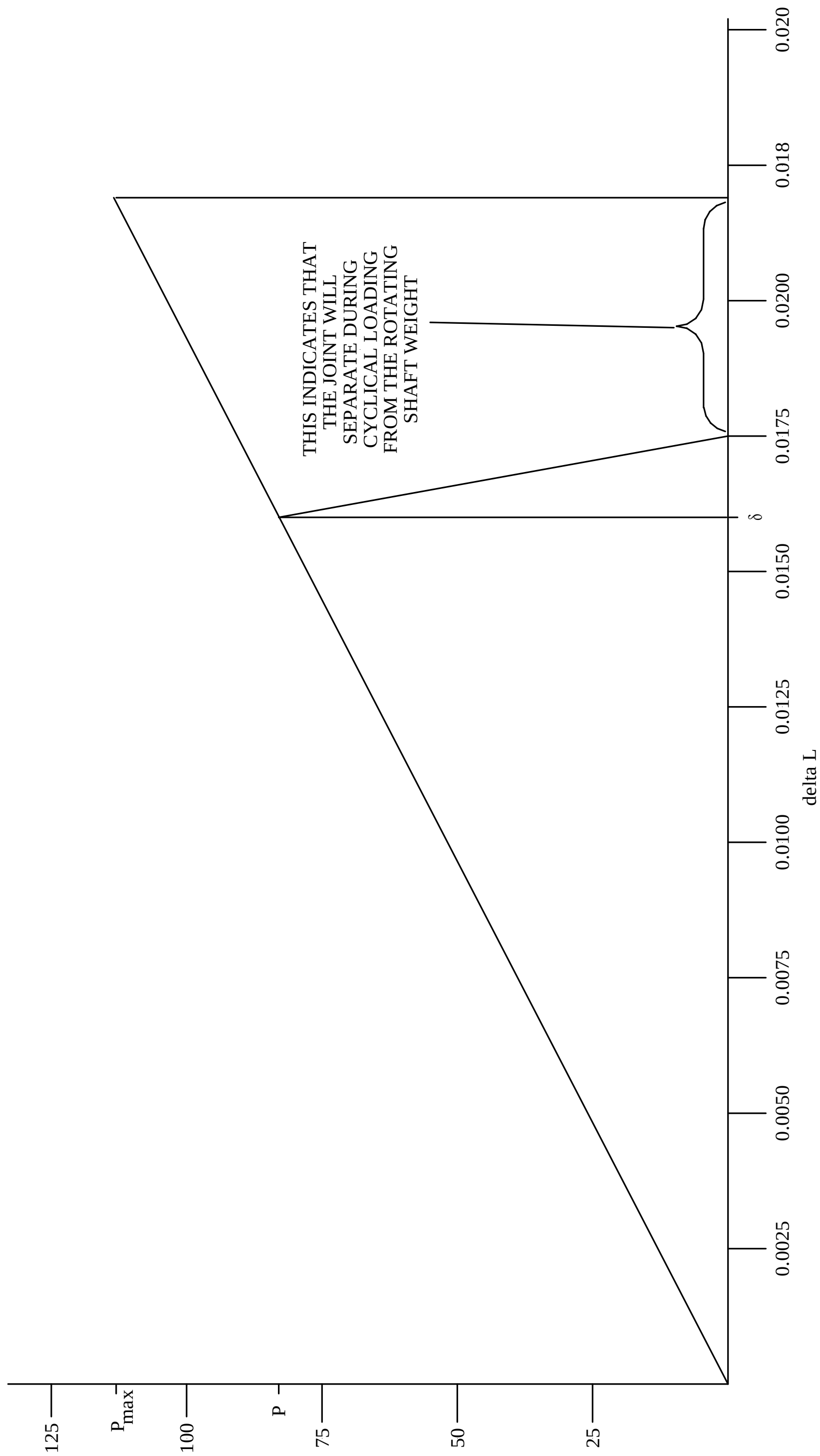
Existing at S_y = 43,500psi



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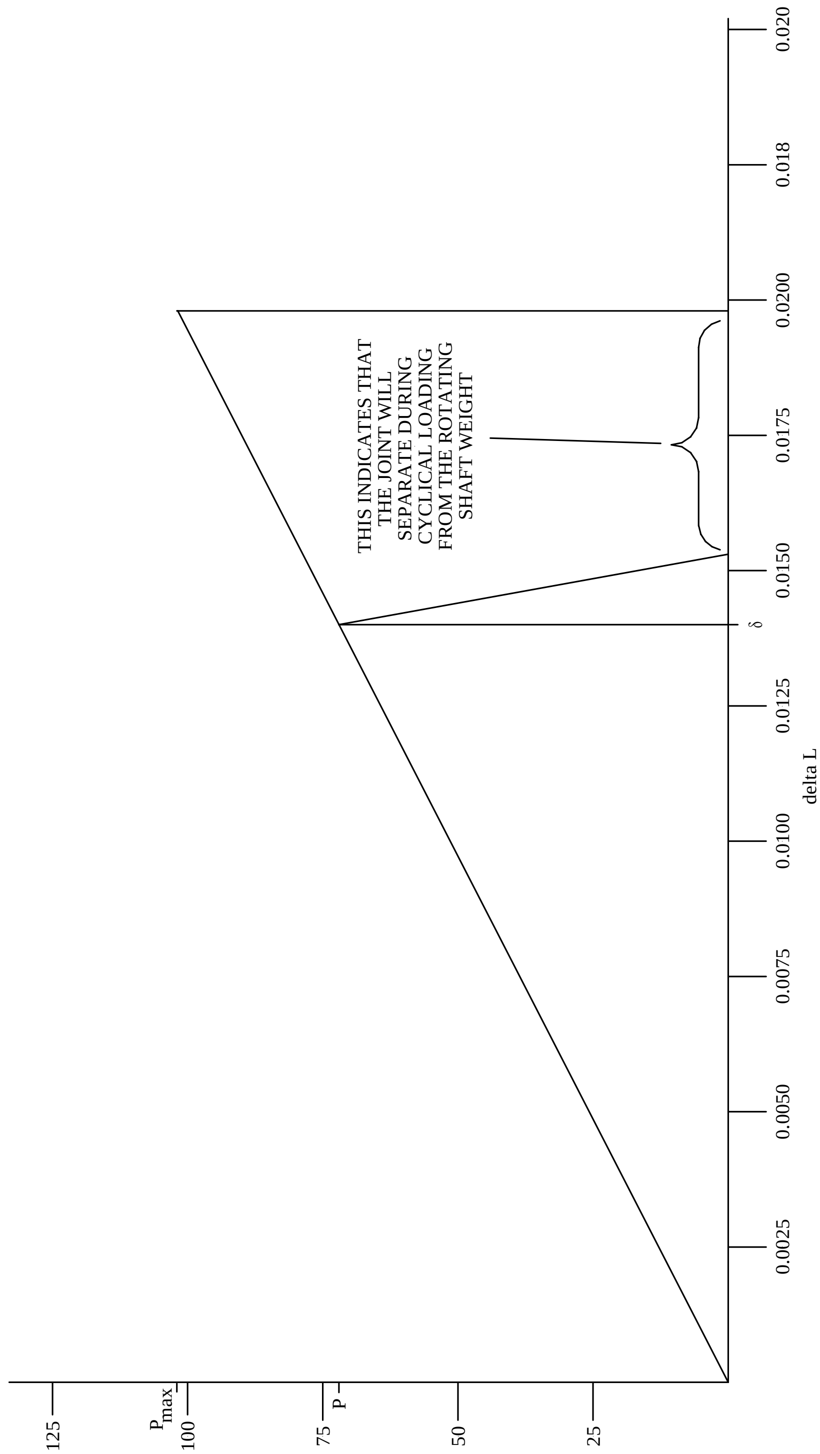
Existing at S_y = 38,000psi



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10. Conclusions

a. Both bolt designs are subject to fatigue failures in areas of stress concentration such as thread roots and head filets. This is consistent with failures to date.
b. Reducing the preload enough to eliminate the fatigue concerns will allow the joint to open up under cyclical loading.
c. The replacement studs will have the same fatigue concerns as the existing design at standard torque (75% of yield strength) and at a reduced stress. They should never be used.